**Registration No.:** | | | | | | | | | | |

**Total Number of Pages: 02**

**Course: M.Sc.I**
**Sub_Code: FMCC703**

**3rd Semester Regular Examination: 2024-25**
**SUBJECT: Advanced Differential Equation**
**BRANCH(S): M.Sc.I (MC)**
**Time: 3 Hours**
**Max Marks: 70**
**Q.Code: R140**
**Answer Question No.1 (Part-I) which is compulsory, any five from rest (Part-II)**
**The figures in the right hand margin indicate marks.**



**Part-I**

**Q1** **Answer the following questions:** **(2 x 10)**

**a)** Write the symmetry property of Hypergeometric function.

**b)** Find the value of Rs. 100 compounded continuously for a period of 10 years at a rate of 4% per year.

**c)** Compute $H_3(x)$ and $H_4(x)$.

**d)** Establish the Rodrigue's formula using Hermite polynomial.

**e)** Determine a fundamental matrix for $x' = Ax$, where $A = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$.

**f)** Define Green's function of Dirichlet problem for Laplace equation.

**g)** Express the Laplace equation in terms of cylindrical coordinates.

**h)** Check whether $u(x, y) = x^2 + y^2$, a solution of Laplace equation.

**i)** Solve the pde $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 1$ by using separation of variable method.

**j)** How method separation of variables is used in solution of heat conduction in a rod of length $l$?

**Part-II**

**Long Answer Type Questions (Answer Any five)**

**Q2** **a)** Show that $e^{2xt} - t^2 = \sum_{n=0}^{\infty} \frac{H_n}{n!} x t^n$. **(5+5)**

**b)** Find the orthogonal trajectories of the family of curves $x^2 + y^2 = 2ax$.

**Q3** **a)** Show that $\frac{d}{dx} F(\alpha, \beta; \gamma; x) = \frac{\alpha\beta}{\gamma} F(\alpha + 1, \beta + 1; \gamma + 1; x)$ in the context of Hypergeometric function. **(5+5)**

**b)** A steam pipe of diameter 1 foot has a cylindrical jacket 6 inches thick made of insulating material (k = 0.00033). If the pipe is kept at $500^0$F and the outside of the jacket at $100^0$F, find the temperature distribution in the jacket.

**Q4** **a)** Find the current in a simple circuit with $C = \infty$ *and* $E(t) = E_0 \sin \omega t$. **(5+5)**

**b)** Find the general solution of the system $\begin{cases} \frac{dx}{dt} = 4x - y \\ \frac{dy}{dt} = 2x + y \end{cases}$.

**Q5** **(5+5)**

**a)** Find the general solution of the system $\begin{cases} \frac{dx}{dt} + 4x + 3y = t \\ \frac{dy}{dt} + 2x + 5y = e^t \end{cases}$.

**b)** Find the Green's function for the first quadrant of the $(x, y)$ −plane.

**Q6** **(5+5)**

**a)** Use operator method solve the system $\begin{cases} 2\frac{dx}{dt} - 2\frac{dy}{dt} - 3x = 0 \\ 2\frac{dx}{dt} + 2\frac{dy}{dt} + 3x + 8y = 0 \end{cases}$.

**b)** Determine the solution of the two-dimensional Laplace equation depending only on $r = \sqrt{x^2 + y^2}$.

**Q7** **a)** Derive D'Alembert's solution of the wave equation. **(5+5)**

**b)** Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ subject to boundary conditions $u(0, t) = u(l, t) = 0, t > 0$ and the initial conditions $u(x, 0) = x$, $l$ being the length of the bar.

**Q8** **a)** A string is stretched and fastened to two points $l$ *apart*. Motion is started by displacing the string in the form $y = a \sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance $x$ from one end at time $t$ is given by $y(x, t) = a \sin\left(\frac{\pi x}{l}\right) a \cos\left(\frac{\pi x}{l}\right)$. **(5+5)**

**b)** Derive the solution of one dimensional heat equation.